Determination of the Roots of the Characteristic Equation for Corrugated and Dielectric Loaded Circular Waveguides

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***Abstract*—An algorithm is developed for the numerical determination of the roots of the characteristic equation for corrugated and dielectric loaded circular waveguides. The algorithm, based on geometrical properties of the characteristic equation and on the identification of the interval of existence of each root, assures a fast and precise determination of all the roots, real or complex, in a given interval.**

I. Introduction

In the analysis and design of microwave devices employing corrugated or dielectric loaded circular waveguides it is necessary to determine, some times repetitively, the roots of the characteristic equation of the waveguides. The analytical expressions for those characteristic equations have been published in the literature several years ago [1]–[3]. In the present paper, geometrical properties of such equations will be employed in order to determine the interval of existence of the roots, allowing the determination, by simple and fast numerical methods, of all the roots, real or complex, in a given interval without any missing roots.

II. Formulation

A. Corrugated Waveguides

The characteristic equation for a corrugated cylindrical waveguide, with internal and external radii a and b, respectively, is given by [1]

$$FC(x) - \chi = 0 \tag{1a}$$

where

$$FC(x) = \frac{k_o a}{x}\left\{\left(\frac{J_m(x)}{x}\right)^2\left(1-\left(\frac{x}{k_o a}\right)^2\right)m^2 - J_m'^2(x)\right\} \cdot \frac{1}{J_m(x)J_m'(x)} \tag{1b}$$

k_0 being the free-space wavenumber, x the desired root, $J_m(x)$ the Bessel function of the first class and of order m, $J_m'(x) = dJ_m(x)/dx$, and χ the normalized susceptance of the corrugation, given by

$$\chi = -\frac{J_m'(k_o a)Y_m(k_o b) - J_m(k_o b)Y_m'(k_o a)}{J_m(k_o a)Y_m(k_o b) - J_m(k_o b)Y_m(k_o a)} \tag{2}$$

$Y_m(x)$ being the Bessel function of the second kind and of order m and $Y_m'(x) = dY_m(x)/dx$.

It was assumed in (1) that the fields have an angular dependence of the form $\cos(m\phi)$ or $\sin(m\phi)$, ϕ being the azimuthal coordinate of the coordinate system with the z axis aligned to the axis of the waveguide.

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For the particular case of imaginary roots, $x = jx_I$, $j = \sqrt{-1}$, (1) takes the form:

$$FI(x_I) = \chi \tag{3a}$$

where

$$FI(x_I) = -\frac{k_o a}{x_I}\left\{\left(\frac{I_m(x_I)}{x_I}\right)^2\left(1+\left(\frac{x_I}{k_o a}\right)^2\right)m^2 - I_m'^2(x_I)\right\} \cdot \frac{1}{I_m(x_I)I_m'(x_I)} \tag{3b}$$

$I_m(x)$ being the modified Bessel function of the first kind and of order m, and $I_m'(x) = dI_m(x)/dx$.

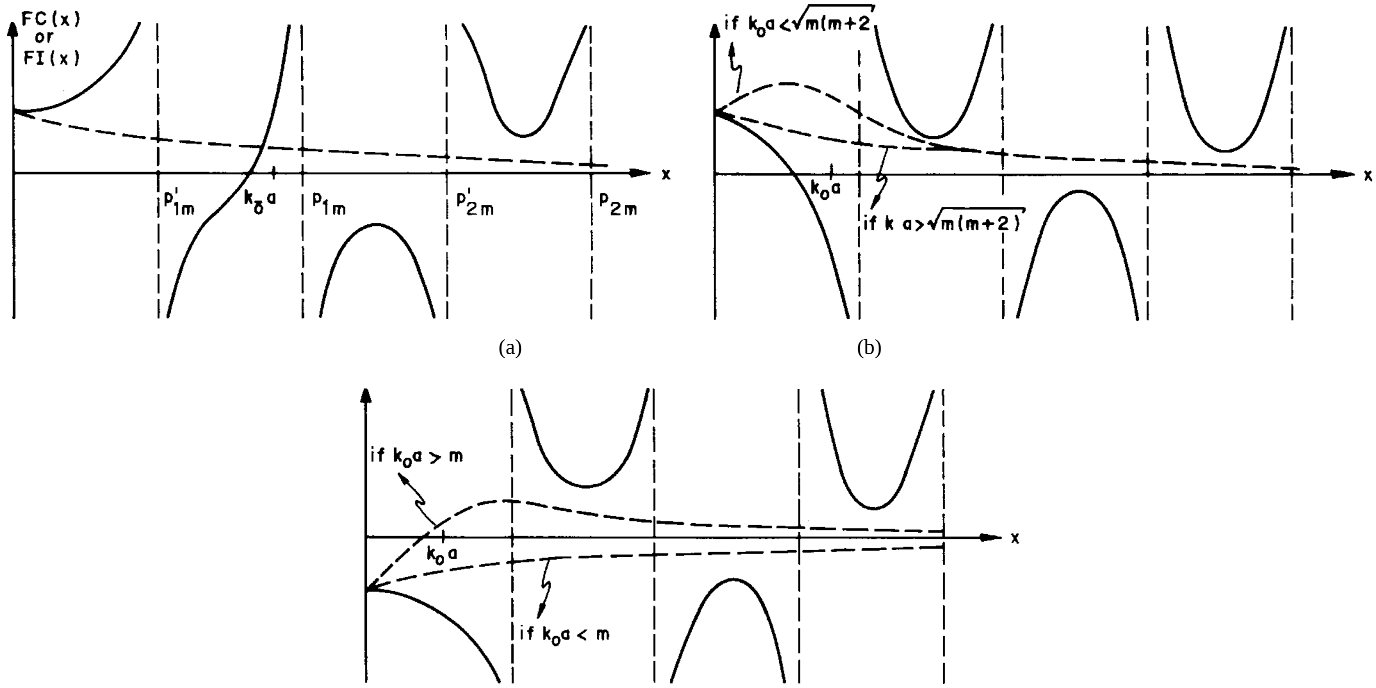
Considering x initially as a real variable, a simple examination of $FC(x)$ and $FI(x_I)$ shows that the following properties are verified:

1) $FC(x)$ has discontinuities at $x = p_{nm}'$ and $x = p_{nm}$, p_{nm}' being the nth root of $J_m'(x)$ and p_{nm} the nth root of $J_m(x)$;
2) at $x = p_{nm}$, $FC(x)$ changes from positive to negative values; at $x = p_{nm}'$, $FC(x)$ changes from positive to negative values if $x < k_o a$, or from negative to positive values if $x > k_o a$;
3) $\lim_{x\to 0} FC(x) = \lim_{x\to 0} FI(x_I) = \frac{(k_o a)^2 - m(m+1)}{(m+1)k_o a}$;
4) $\lim_{x\to\infty} FI(x_I) = \frac{(k_o a)^2 - m^2}{(k_o a)x}$;
5) In the neighborhood of $x = 0$, $FI(x_I)$ is a decreasing function of x_I if $k_o a > \sqrt{m(m+2)}$, or an increasing function if $k_o a < \sqrt{m(m+2)}$.

Plots of $FC(x)$ and $FI(x_I)$ illustrating the above properties, are shown in Fig. 1, the cases $k_o a > p_{nm}'$, $p_{1m}' > k_o a > \sqrt{m(m+2)}$, $\sqrt{m(m+2)} > k_o a > \sqrt{m(m+1)}$, $\sqrt{m(m+1)} > k_o a > m$, and $m > k_o a$ being considered.

The properties of $FC(x)$ and $FI(x_I)$ and the plots shown in Fig. 1(a)–(c), permit the identification of the intervals of existence of the roots of the characteristic equation, as indicated below:

1) *for* $0 < x < p_{1m}'$:
   • if $k_o a > p_{1m}'$—there will be one real root if $FC(0) < \chi$; one imaginary root if $FC(0) > \chi$ and $\chi > 0$; no root if $\chi < 0$;
   • if $p_{1m}' > k_o a > \sqrt{m(m+2)}$—there will be no root if $FC(0) < \chi$; one real and one imaginary root if $FC(0) > \chi$ and $\chi > 0$; one real root if $\chi < 0$;
   • if $\sqrt{m(m+2)} > k_o a > \sqrt{m(m+1)}$—there will be no root if $\max(FI(x)) < \chi$; two imaginary roots if $\max(FI(x)) > \chi$ and $FC(0) < \chi$; one real and one imaginary root if $FC(0) > \chi$ and $\chi > 0$; one real root if $\chi < 0$;
   • if $\sqrt{m(m+1)} > k_o a > m$—there will be no root if $\max(FI(x)) < \chi$; two imaginary roots if $\max(FI(x)) > \chi$ and $\chi > 0$; one imaginary root if $FC(0) < \chi$ and $\chi < 0$; one real root if $FC(0) > \chi$;
   • if $k_o a < m$—there will be no root if $\chi > 0$; one imaginary root if $FC(0) < \chi$ and $\chi < 0$; one real root if $FC(0) > \chi$;
2) *for* $p_{1m}' < x < p_{n_1 m}$:
   • where $p_{n_1 m}$ is the n_1th root of $J_m(x)$, satisfying the condition that $p_{n_1 m}'$ is the largest root of $J_m'(x)$ smaller than $k_o a$: there will be one real root on each interval $[p_{1m}', p_{1m}]$, $[p_{1m}, p_{2m}'], \cdots [p_{n_1 m}', p_{n_1 m}]$;
3) *for* $x > p_{n_1 m}$:
   • if $\chi > 0$, there will be two real roots or a pair of complex conjugate roots on each interval $[p_{n_1+1,m}', p_{n_1+1,m}]$, $[p_{n_1+2,m}', p_{n_1+2,m}]$, $\cdots$; if $\chi < 0$ there will be two real



(a) (b)

(c)

Fig. 1. Plot of the functions $FC(x)$ (——) and $FI(x)$ (- - - -), for the following cases: (a) $k_o a > p'_{1m}$. (b) $p'_{1m} > k_o a > \sqrt{m(m+2)}$ and $\sqrt{m(m+2)} > k_o a > \sqrt{m(m+1)}$. (c) $\sqrt{m(m+1)} > k_o a > m$ and $m > k_o a$.

roots or a pair of complex conjugate roots on each interval $[p_{n_1 m}, p'_{n_1+1,m}]$, $[p_{n_1+1,m}, p'_{n_1+2,m}] \cdots$.

Once the intervals of existence of the roots are identified, their values can be determined in a fast and simple way.

In the case where there is only real root in a given interval, the limits of the interval, and the sign of the function $FC(x)$ on these limits are known. The root can be calculated by the method of [4], using as interval for the search of the root $[xi + \Delta x, xf - \Delta x]$, xi and xf being the lower and upper limits of the interval of existence of the root and Δx a small number. If the sign of $FC(xi + \Delta x)$ is different from the previously known for $\lim_{dx \to 0} FC(xi + dx)$, the value of the root can be established as $xi + \Delta x/2$ with an error less then $\Delta x/2$ and the same reasoning applying to the upper limit of the interval.

In the case where there are two real or a pair of complex conjugate roots in a given interval, the minimum of $FC(x)/\chi$ in the interval, xm, is initially determined numerically. If $FC(xm)/\chi < 1$, there will be two real roots, if not, there will be a pair of complex conjugate roots. The real roots can determined by the method of [4], considering the intervals $[xi + \Delta x, xm]$ and $[xm, xf - \Delta x]$ for the search of the roots. For one of the complex roots (the second is the complex conjugate), the method of [5] can be applied.

B. Dielectric Loaded Waveguides

The characteristic equation for a dielectric loaded circular waveguide, composed of an external metallic wall of radius b and a concentric dielectric cylinder of radius a, is given by [3]:

$$FD(k_{c1}a) = \frac{m^2\beta^2}{k_o^2}\left[\frac{(k_{c2}a)^2 - (k_{c1}a)^2}{(k_{c2}a)^2}\right]^2 - \left[FJ(k_{c1}a) - \frac{R}{(k_{c2}a)^2}(k_{c1}a)^2\right] \cdot \left[\epsilon_{r1} FJ(k_{c1}a) - \epsilon_{r2}\frac{S}{(k_{c2}a)^2}(k_{c1}a)^2\right] = 0 \quad (4)$$

$k_{c1}a$ being the desired root, k_o the free-space wavenumber, ϵ_{r1} the dielectric constant of the dielectric cylinder, ϵ_{r2} the dielectric constant of the region external to the dielectric cylinder $\beta^2 = \epsilon_{r1}k_o^2 - k_{c1}^2 = \epsilon_{r2}k_o^2 - k_{c2}^2$, and

$$FJ(k_{c1}a) = k_{c1}a\frac{J'_m(k_{c1}a)}{J_m(k_{c1}a)} \quad (5a)$$

$$R = \frac{J'_m(k_{c2}a)Y'_m(k_{c2}b) - J'_m(k_{c2}b)Y'_m(k_{c2}a)}{J_m(k_{c2}a)Y'_m(k_{c2}b) - J'_m(k_{c2}b)Y_m(k_{c2}a)}k_{c2}a \quad (5b)$$

$$S = \frac{J'_m(k_{c2}a)Y_m(k_{c2}b) - J_m(k_{c2}b)Y'_m(k_{c2}a)}{J_m(k_{c2}a)Y_m(k_{c2}b) - J_m(k_{c2}b)Y_m(k_{c2}a)}k_{c2}a. \quad (5c)$$

If $k_{c1}a$ is real and $k_{c1} < k_o\sqrt{\epsilon_{r1} - \epsilon_{r2}}$, k_{c2} will be imaginary and the Bessel functions, which appear in the expressions of the functions R and S must be replaced by the corresponding modified Bessel functions. If $m = 0$, (4) decomposes into independent TE and TM modes.

In the development below, it will be considered $\epsilon_{r1} > \epsilon_{r2}$. The case $\epsilon_{r1} < \epsilon_{r2}$ can be easily adapted.

In a similar way as done for corrugated waveguides considering $x = k_{c1}a$ initially as a real variable the following properties of $FD(x)$ are verified:

1) $FD(x)$ has discontinuities at $x = p_{nm}$, $x = k_o a\sqrt{\epsilon_{r1} - \epsilon_{r2}}$, $x = xr_{nm}$ and $x = xs_{nm}$, xr_{nm} and xs_{nm} being the values of x for which $k_{c2}a$ is the nth root of the denominator of the functions R and S, respectively. The numerical determination of xr_{nm} and xs_{nm} is in the Appendix.
2) The sign of $FD(x)$ is negative immediately before and immediately after the discontinuity at $x = p_{nm}$.
3) $FD(x)$ has opposite signs immediately before and immediately after the discontinuities at $x = xr_{nm}$ and $x = xs_{nm}$. The sign of $FD(x)$ immediately after xr_{nm} can be determined by calculating $FD(xr_{nm})$, but replacing in the denominator or

TABLE I
VALUES OF THE FIRST THREE ROOTS OF THE CHARACTERISTIC EQUATION, χ AND $k_o a$, FOR $m = 1$, FOR A CORRUGATED CYLINDRICAL WAVEGUIDE WITH $a = 28$ mm AND $b = 70$ mm

Freq(GHz)	x1	x2	x3	χ	$k_o a$
2.3	2 .5040+j1.8820	2.5040-j1.8820	5.9165+j.67310	.171	1.35
2.4	j1.2452	j11.229	5.4515+j.92349	.0531	1.41
2.5	1.1725	5.0688+j.70080	5.0688-j.70088	-.0715	1.47
2.6	1.5069	4.9226+j.45328	4.9226-j.45328	-.206	1.52
2.7	1.6506	4.8465+j.17573	4.8465-j.17573	-.355	1.58
2.8	1.7309	4.5025	5.0925	-.524	1.64
2.9	1.7809	4.3444	5.1786	-.722	1.70
3.0	1.8135	4.2399	5.2252	-.963	1.76
3.1	1.8350	4.1597	5.2555	-1.27	1.82
3.2	1.8486	4.0928	5.277	-1.69	1.88

R, xr_{nm} by $xr_{nm} + \Delta x$ with Δx being a small number. The same reasoning applies to the discontinuities at $x = xs_{nm}$.

4) $\lim_{x\to 0} FD(x) = 0$ and $FD(x)$ assumes positive values in the neighborhood of $x = 0$.

As a consequence of the above properties, in each interval between two consecutive discontinuities there will be:

1) one real root if the signs of $FD(x)$ at the lower and upper limits of the interval are opposite;
2) two real roots if $FD(x)$ has the same sign at both limits of the interval and $\min[FD(x)\cdot \text{sign}(FD(x_i))] < 0$, $\text{sign}(FD(x_i))$ being the sign of $FD(x)$ at the lower limit of the interval;
3) a pair of complex roots if $FD(x)$ has the same sign at both limits of the interval and $\min[FD(x)\cdot \text{sign}(FD(x_i))] > 0$.

The real and complex roots can be determined applying the same procedures used in the case of corrugated waveguides.

III. NUMERICAL RESULTS

Computer programs were elaborated according to the methods described above to calculate the roots of the characteristic equations for corrugated and dielectric loaded waveguides.

The values of the first three roots and the corresponding values of χ and $k_o a$ for a corrugated cylindrical waveguide with $a = 28.0$ mm, $b = 70.0$ mm, at different frequencies, for $m = 1$, are shown in Table I. The table illustrates the several ranges of values of $k_o a$ discussed in item 2. Values of the propagation constant of the first two modes of propagation (EH_{11} and HE_{11}) for this waveguide, at different frequencies, are shown in Table II. As can be observed, the first mode has two propagation constants, the first one corresponding to a backward wave, and the second one to a slow wave. This kind of behavior is indicated in [1, Fig. 2(c)] that shows the dispersion diagram for a corrugated waveguide with the same dimensions of the example considered here.

The first four roots of the characteristic equation for a dielectric loaded circular waveguide with $a = 10.0076$ mm, $b = 12.70$ mm, $\epsilon_{r1} = 37.6$, $\epsilon_{r2} = 1.0$, at the frequency of 4.0 GHz, for $m = 0$, 1, 2, 3, and 4, are shown in Table III. The roots marked with * are the same as on [6, Table II]. Complex roots were not considered in this reference.

IV. CONCLUSION

The interval of existence of each root of the characteristic equation for corrugated and dielectric loaded circular waveguides can be determined from the geometrical properties of these equations. Once the intervals of existence are known, the roots can be calculated by simple and reliable numerical methods.

TABLE II
VALUES OF THE PROPAGATION CONSTANT OF THE FIRST TWO MODES WITH $m = 1$, FOR A CORRUGATED CYLINDRICAL WAVEGUIDE WITH $a = 28$ mm, $b = 70$ mm

Freq(GHz)	β (EH_{11} mode)	β (HE_{11} mode)
2.37	102.208; 175.296	
2.40	67.113; 404.158	
2.44	48.462; 6074.48	
2.48	36.399	
2.52	26.858	
2.56	18.211	
2.60	8.297	
3.15		4.672
3.65		38.787
4.15		62.374
4.65		97.181

TABLE III
FIRST FOUR ROOTS OF THE CHARACTERISTIC EQUATION, FOR A DIELECTRIC LOADED CIRCULAR WAVEGUIDE WITH $a = 10.0076$ mm, $b = 12.70$ mm, $\epsilon_{r1} = 37.6$, $\epsilon_{r2} = 1.0$, AT 4.0 GHz, FOR $m = 0, 1, 2, 3$, AND 4

m	1st root	2nd root	3rd root	4th root
0	3.25428	3.7863	5.0962	5.8988
1	2.2607*	4.4052*	5.2145*	5.3523*
2	3.6865*	5.0922*	5.8560*	6.6960*
3	5.2456+j.3337	5.2456-j.3337	6.7953*	8.0237
4	6.0853+j.66108	6.0853-j.66108	7.7699	9.3089

APPENDIX

Roots of the Denominators of the Functions R and S

If k_{c2} is imaginary, the denominators of R and S in (5b) and (5c) have no roots. If k_{c2} is real, the roots of the denominator of R are the solutions of:

$$J_m(\delta x)Y_m'(x) - J_m'(x)Y_m(\delta x) = 0 \quad \text{(A1)}$$

where $\delta = a/b$, and $x = k_{c2}b$

Applying the asymptotic expressions of the Bessel functions for small and large arguments, the following approximate expressions results for the nth root of (A1):

$$\begin{aligned} xr_{nm}^{<} &= p'_{nm},\ n = 1, 2, \cdots, && \text{if } m > 0,\ \delta \ll 1 \\ xr_{1m}^{<} &= \sqrt{\frac{-2}{\ln(\delta)}}, && \text{if } m = 0,\ \delta \ll 1 \\ xr_{nm}^{<} &= p'_{n-1,0},\ n = 2, 3, \cdots, && \text{if } m = 0,\ \delta \ll 1 \\ xr_{nm}^{>} &= \frac{(2n-1)\pi/2}{(1-\delta)},\ n = 1, 2, \cdots, && \text{if } \delta \cong 1 \end{aligned}$$

$xr_{nm}^{<}$ and $xr_{nm}^{>}$ being the approximate nth root of (A1) for the cases $\delta \ll 1$ and $\delta \cong 1$, respectively.

Based on the above results, the following semiempirical expression, valid for $0 \le m \le 50$, $0 < \delta < 1$, can be built for the approximate values of the first three roots of (A1):

$$xr_{nm}^{ap} = \sqrt{(c_1 xr_{nm}^{<})^2 + (c_2 xr_{nm}^{>})^2}, \quad n = 1, 2, 3; \quad 0 \le m \le 50 \quad \text{(A2)}$$

where

$$c_1 = (1-\delta)^{\alpha r_n}, \quad c_2 = \left(\frac{2\delta}{1+\delta}\right)^{\beta r_n (m+1)^{\gamma r_n}}$$

with

$$\alpha r_n = \begin{cases} 0.088\,969, & \text{if n=1} \\ 0.011\,57, & \text{if n=2} \\ 0.019\,70, & \text{if n=3} \end{cases}$$

$$\beta r_n = \begin{cases} 0.4450, & \text{if n=1} \\ 0.3985, & \text{if n=2} \\ 0.4277, & \text{if n=3} \end{cases}$$

$$\gamma r_n = \begin{cases} 0.6189, & \text{if n=1} \\ 0.6039, & \text{if n=2} \\ 0.5178, & \text{if n=3.} \end{cases}$$

The values of the parameters αr_n, βr_n, and γr_n were determined numerically, in order to minimize the average error of the approximate roots.

The exact values of the roots can now be calculated by the method of [4] using as interval for the search of the roots xr_{nm}: $[xr_{1m}^{ap} - .4(xr_{2m}^{ap} - xr_{1m}^{ap}), xr_{1m}^{ap} + .4(xr_{2m}^{ap} - xr_{1m}^{ap})]$, for $n = 1$, $[xr_{nm}^{ap} - .4(xr_{nm}^{ap} - xr_{n-1,m}), xr_{nm}^{ap} + .4(xr_{nm}^{ap} - xr_{n-1,m})]$, for $n = 2$ and $n = 3$, and $[xr_{n-1,m} + .4(xr_{n-1,m} - xr_{n-2,m}), xr_{n-1,m} + 1.4(xr_{n-1,m} - xr_{n-2,m})]$ for $n \geq 4$.

The roots of the denominator of the function S are the solution of:

$$J_m(\delta x)Y_m(x) - J_m(x)Y_m(\delta x) = 0. \quad \text{(A3)}$$

It should be observed that this equation is the same as the characteristic equation for TM modes in a coaxial circular waveguide.

The procedure to determine the roots xs_{nm} is the same as applied to the function R. The method of [4] is again used, with the same intervals defined above for R, but replacing xr_{nm}^{ap} and xr_{nm} by xs_{nm}^{ap} and xs_{nm}, respectively. The values of xs_{nm}^{ap} are given by:

$$xs_{nm}^{ap} = \sqrt{(c_3 xs_{nm}^{<})^2 + (c_4 xs_{nm}^{>})^2}, \quad n = 1, 2, 3, \quad 0 \leq m \leq 50 \quad \text{(A4)}$$

with

$$c_3 = (1. - \delta)^{\alpha s_n} \quad c_4 = \left(\frac{2\delta}{1+\delta}\right)^{\beta s_n (m+1)^{\gamma s_n}}$$
$$xs_{nm}^{<} = p_{nm} \qquad xs_{nm}^{>} = \frac{n\pi}{1+\delta}$$

$$\alpha s_n = \begin{cases} -0.002\,591, & \text{if n=1} \\ 0.015\,33, & \text{if n=2} \\ 0.024\,62, & \text{if n=3} \end{cases}$$

$$\beta s_n = \begin{cases} 0.2853, & \text{if n=1} \\ 0.4413, & \text{if n=2} \\ 0.4068, & \text{if n=3} \end{cases}$$

$$\gamma s_n = \begin{cases} 0.8402, & \text{if n=1} \\ 0.5396, & \text{if n=2} \\ 0.5014, & \text{if n=3.} \end{cases}$$

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Analytical Behavior of the Noise Resistance and the Noise Conductance for a Network with Parallel and Series Feedback

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An analysis is presented of the changes of the noise parameters of a two-port network when noisy series and parallel feedback immittances are applied. Exact formulas for the noise parameters R_n, g_n, and ρ_n are given as functions of the feedback for a given network. It is proved that R_n always reaches a minimum when a reactive series feedback is considered. The same results are demonstrated for g_n since a duality principle is pointed out. The results are valid for a wide range of linear microwave two-port networks, either passive or active, and they are used to confirm the data from previously published work.

***Index Terms*—Amplifier noise, feedback amplifiers, feedback circuits, microwave amplifiers, noise.**

I. INTRODUCTION

In [1], some guidelines are outlined for feedback amplifier design. The resistive parallel feedback has been investigated by [2] and [3]. The change of the noise figure in the case of either parallel or series feedback was worked out by [4]. In [5], series and parallel feedback are analyzed in order to get simultaneously optimum noise and good input/output standing-wave ratio (SWR). In [6], monolithic technology to fabricate a series feedback amplifier in order to get good repeatability during fabrication and the simultaneous noise match and optimum input SWR is applied. Both simulation and experimental validation of an *X*-band monolithic four-stage low-noise amplifier with series feedback is carried out in [7]; however, the paper does not detail how the simulation has been carried out.

This paper generalizes the results of [6] and [7] using a procedure similar to [1], provides a mathematical tool to investigate the signal

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